Velocity of sound in two-phase mixtures

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Velocity of sound is computed for water, ammonia, Freon-12 and isobutane in two-phase mixtures in thermodynamic equilibrium. A computational method is developed which yields the sound velocity in terms of the thermodynamic coordinates of the substance (*T*, *x*) without the use of diagrams. Corresponding velocities of sound for the four substances considered exhibit a certain similarity which is examined statistically. The relationship between the sound velocity and the critical mass flux is also investigated.

Key words: *Sound velocity, two-phase mixtures, critical mass flux*

In single-phase flow the sonic velocity is important because it yields the critical mass flux of the substance and the speed of propagation of small disturbances. In addition, the sonic velocity determines the Mach number, one of the most important dimensionless numbers in transonic and supersonic flow.

In two-phase mixtures it seems that the sonic velocity and the critical mass flux are barely related; the relationship between the sonic velocity and the propagation of small pressure pulses is still undetermined. Other phenomena, such as the inhomogeneity of the mixture and the thermodynamic equilibrium (or the absence of it) become important when two-phase flow is considered. These effects would complicate any experimental investigation of the influence on the sonic velocity in two-phase mixtures of an evaporative component.

There have been some analytical investigations of the velocity of sound primarily in steam-water systems[1-3]. Others[4-6] have determined the propagation velocity of the pressure pulses; it appears from the studies that the latter quantity is different from the velocity of sound. Attenuation and dispersion of sound in two-phase mixtures has also been studied analytically[7]. A good survey of the work performed in this topic was given by Gouse and Brown[8] in 1964 and recent discussions of the subject are by Hsu and Graham[9] and Delhaye *et al*[10].

All the previous studies concerned with the velocity of sound derive this quantity from geometric relations in the Mollier diagram. Here, it will be attempted to derive the sound velocity from purely thermodynamic relations. Thus, the velocity of sound will be given as a function of the state of the fluid; this would enable us to calculate it for systems where Mollier diagrams are not readily available. It is understood that the degree of dispersion will also play an important role in the determination of the sound velocity. Here highly dispersed flow situations are examined, which are compatible with the homogeneous flow assumption stated later.

The substances considered are water, ammonia, Freon-12 and isobutane. The method of calculation is derived from thermodynamic principles and may be applied to any two-phase evaporative system. It appears that for these four substances, the velocity of sound exhibits certain similarities, which are further explored. Finally, the relationship between critical flow and the velocity of sound is examined based on experimental data for the critical flow.

Development of the working equation

Throughout the calculations, the following principal assumptions are made:

- The two-phase mixture is homogeneous in phase composition, with no slip between the phases. Hence the density of the mixture can be written as:

$$\rho = \frac{1}{v} = \frac{1}{v'(1-x)+v''x} \tag{1}$$

- The two phases are in thermodynamic equilibrium. This implies that the temperature and pressure of the two phases are the same.

These main assumptions will be supplemented with others pertaining only to parts of the work.

Equations for the velocity of sound

The velocity of sound is defined as:

$$\alpha = \left(\frac{\partial P}{\partial \rho}\right)_s^{1/2} \tag{2}$$

or, upon substituting $\rho = 1/v$;

$$\alpha = v\left(-\left(\frac{\partial P}{\partial v}\right)_s\right)^{1/2} \tag{3}$$

The derivative $(\partial P/\partial v)_s$ will be evaluated by thermodynamic transformations relating the appropriate variables. The objective is to reduce the expression for the velocity of sound in the form, $\alpha = \alpha(T, x)$ so that it would be calculated directly for any state of the two-phase mixture. Eq (3) can be written:

$$\alpha^2 = v^2 \left(\frac{\partial s}{\partial v}\right)_P \left(\frac{\partial P}{\partial s}\right)_v \tag{4}$$

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Using the Maxwell relation $(\partial P/\partial s)_v = -(\partial T/\partial v)_s$ and the chain rule for differentiation, the previous equations yield:

$$\alpha^2 = v^2 \left(\frac{\partial s}{\partial v}\right)_P \left(\frac{\partial s}{\partial v}\right)_T \left(\frac{\partial T}{\partial s}\right)_v \tag{5}$$

The thermodynamic equilibrium in the two-phase region implies that the Clausius–Clapeyron equation is valid and, hence, pressure is a function only of temperature. Therefore, the partial derivatives at constant pressure are the same with those at constant temperature.

$$\left(\frac{\partial s}{\partial v}\right)_P = \left(\frac{\partial s}{\partial v}\right)_T = \frac{s'' - s'}{v'' - v'} = \frac{s_{fg}}{v_{fg}} = \frac{dP}{dT} \tag{6}$$

Hence:

$$\alpha^2 = v^2 \left(\frac{dP}{dT}\right)^2 \left(\frac{\partial T}{\partial s}\right)_v \tag{7}$$

It seems that in Eq (7) it is only the derivative $(\partial T/\partial s)_v$ that cannot be evaluated directly in terms of the thermodynamic coordinates T and x. In the previous attempts to calculate the velocity of sound[1,2], this expression was obtained graphically from a Mollier or a T, s diagram, along a constant volume line. This procedure may introduce substantial errors in the calculations, especially near the liquid saturation line. Furthermore, it does not allow the calculation of the velocity of sound for substances where such diagrams are not available. Other thermodynamic relations will be developed here in order to calculate that derivative.

The differential form of the specific volume v of the fluid is:

$$dv = dv' + x\, dv_{fg} + v_{fg}\, dx \tag{8}$$

and hence, for a constant volume process:

$$dx = -\frac{1}{v_{fg}}(dv' + x\, dv_{fg}) \tag{9}$$

Similarly the differential ds for the specific entropy of the two-phase mixture is:

$$ds = ds' + s_{fg}\, dx + x\, ds_{fg} \tag{10}$$

Substituting for dx from Eq (9) we obtain for the constant volume process:

$$ds = ds' - \frac{s_{fg}}{v_{fg}}(dv' + x\, dv_{fg}) + x\, ds_{fg} \tag{11}$$

For the saturated liquid $ds' = c'\, dT/T$ where c' is the derivative dh'/dT evaluated at the saturation liquid line. This quantity is only a function of the temperature since the dryness fraction at this line is zero. Thus, the derivative $(\partial s/\partial T)_v$ can be written as:

$$\left(\frac{\partial s}{\partial T}\right)_v = \frac{c'}{T} + x\frac{ds_{fg}}{dT} - \frac{s_{fg}}{v_{fg}}\left(\frac{dv'}{dT} + x\frac{dv_{fg}}{dT}\right) \tag{12}$$

Using the Clausius–Clapeyron Eq (6) for s_{fg} and its derivatives, the above equation becomes:

$$\left(\frac{\partial s}{\partial T}\right)_v = \frac{c'}{T} + xv_{fg}\frac{d^2P}{dT^2} - \frac{dP}{dT}\frac{dv'}{dT} \tag{13}$$

or:

$$\left(\frac{\partial s}{\partial T}\right)_v = \frac{c'}{T} + xv_{fg}\frac{d}{dT}\left(\frac{s_{fg}}{v_{fg}}\right) - \frac{s_{fg}}{v_{fg}}\frac{dv'}{dT} \tag{14}$$

Depending on the available data, either Eq (13) or Eq (14) may be used with Eq (7) to calculate the isentropic velocity of sound. It must be emphasized that all the quantities, P, c', v_{fg}, s_{fg} and v' are functions of temperature only in the saturation region. Therefore, it is simple to obtain their functional form, either from a fundamental equation or from correlations fitted to available data.

Velocity of sound

The final expressions for the velocity of sound are:

$$\alpha^2 = \frac{v^2\left(\frac{dP}{dT}\right)^2}{\frac{c'}{T} + xv_{fg}\frac{d^2P}{dT^2} - \frac{dP}{dT}\cdot\frac{dv'}{dT}} \tag{15}$$

or:

$$\alpha^2 = \frac{\frac{v^2}{T^2}\left(\frac{h_{fg}}{v_{fg}}\right)^2}{\frac{c'}{T} + xv_{fg}\frac{d}{dT}\left(\frac{h_{fg}}{Tv_{fg}}\right) - \frac{h_{fg}}{Tv_{fg}}\frac{dv'}{dT}}. \tag{16}$$

If an accurate expression for the saturated pressure exists, Eq (15) is appropriate. Otherwise correlations for h_{fg}, v_{fg}, c' and v' are obtained and used in Eq (16). In the latter, it is found that it is better to use a latent heat (h_{fg}) expression, rather than entropy (s_{fg}).

For water, Eq (15) was used with an expression for the saturation pressure obtained from

Nomenclature

c	Heat capacity
G	Mass flux
h	Enthalpy
P	Pressure
s	Entropy
T	Temperature
v	Specific volume
x	Dryness fraction
α	Velocity of sound
ε	Dimensionless velocity of sound
ρ	Density

Subscripts

cr	Critical
fg	Latent
r	Reduced

Superscripts

′	Liquid
″	Vapour

the Keenan *et al*[11] tables. This expression is claimed to have maximum deviation of 1 part in 6000 when compared with data[12]. The other quantities c', v_{fg} and v' were calculated from correlations fitted to the data of these tables. These correlations have maximum deviation less than 1.0%. The calculated velocity of sound a is shown in Fig 1(a) as a function of temperature, T, for different dryness fractions x. It is seen that a is substantially lower for smaller x and that the curves converge at the critical point. The values of a obtained here agree well with all the values obtained by other methods[1,2,8].

Next the velocity of sound for ammonia was calculated using Din's data[13]. In this case, Eq (16) was used for the calculations. The results are given in Fig 1(b), where it is seen that the velocity of sound shows the same behaviour, qualitatively, as water.

Freon-12 was the next substance investigated; data for its thermophysical properties are obtained from Refs 14 and 15. The results are given in Fig 1(c).

Finally, isobutane was examined using Goodwin's data[16]. The velocity of sound was calculated from Eq (16) and is shown in Fig 1(d).

Details of the calculations, the functions used for the correlation of data and tables with the exact values of the sound velocity are given elsewhere[17]. At first, it seems that the low isentropic velocity of sound near the saturated liquid states is incompatible with common experience that sound velocities are high for liquids. A moment's reflection, however, proves that the states examined here are to the right of the saturation curve and the presence of a small quantity of vapour increases drastically the compressibility of the mixture, thus making a small. This result is confirmed by other studies[1,2,8] for water. In fact, the function a is discontinuous at both boundaries of the saturation curve ($x = 0$ and $x = 1$), because the compressibility of any substance is discontinuous across the saturation line.

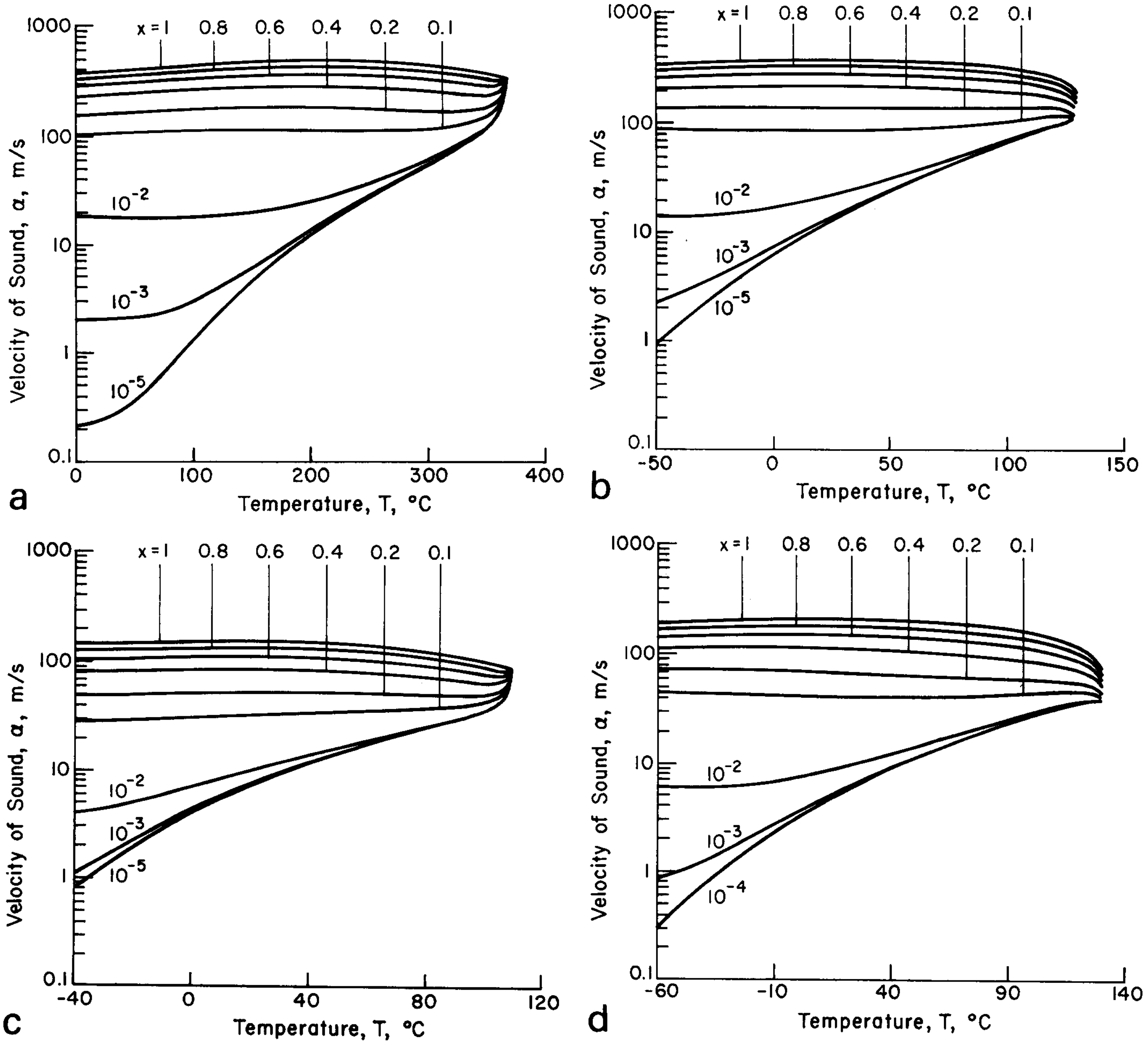

Fig 1 Isentropic velocity of sound in the two-phase region for (a) water, (b) ammonia, (c) Freon-12, and (d) isobutane

Similarity

It is observed in Fig 1 that the form of the curves for the velocity of sound is the same for the substances examined. Although the actual values of α may change, the curves of the function $\alpha(T, x)$ appear similar in shape. This leads us to think that a similarity may exist for the velocity of sound. In order to examine this possibility the function $\alpha(T, x)$ is calculated in a dimensionless form by dividing with $(h_{fg})^{1/2}$. The temperature T is also made dimensionless by using the reduced temperature $T_r = T/T_{cr}$. Then Eq (16) becomes:

$$\varepsilon^2(T_r, x) = \frac{\alpha^2(T, x)}{h_{fg}} = \frac{\dfrac{v^2}{T_r^2}\left(\dfrac{h_{fg}}{v_{fg}^2}\right)}{\dfrac{c'}{T_r}T_{cr} + x\,d_{fg}\dfrac{d}{dT_r}\left(\dfrac{h_{fg}}{T_r v_{fg}}\right) - \dfrac{h_{fg}}{T_r v_{fg}}\dfrac{dv'}{dT_r}} \qquad (17)$$

For the computations of $\varepsilon(T_r, x)$ the values of T_r were taken to be 0.55 and above to ensure that the substances are above their triple points. The results are given in Tables 1, 2, 3 and 4 for water, ammonia, Freon-12 and isobutane respectively. In these tables the dryness fractions vary from 0.1 to 1.

A statistical analysis of these results was undertaken and the average and standard deviation of the corresponding entries in the four tables are calculated. The results of this calculation are shown in Table 5. Here, the average of the four values is written and underneath it, in parenthesis, the standard deviation. It can be seen that the standard deviation is always less than 10% and in most cases less than 7% of the average. The higher deviations occur near the critical point ($T_r = 0.95$) or near the triple point (low T_r). These results suggest that the dimensionless velocity of sound, which is a state property of the mixture, is almost the same for corresponding states of the four substances examined.

Table 1 Dimensionless velocity of sound for water $\varepsilon(T_r, x)$

	x					
T_r	0.1	0.2	0.4	0.6	0.8	1.0
0.55	0.074	0.115	0.172	0.215	0.251	0.283
0.60	0.077	0.121	0.181	0.229	0.268	0.302
0.65	0.078	0.125	0.191	0.241	0.283	0.319
0.70	0.080	0.128	0.198	0.251	0.295	0.333
0.75	0.081	0.130	0.202	0.258	0.304	0.344
0.80	0.084	0.132	0.206	0.263	0.311	0.353
0.85	0.090	0.136	0.209	0.267	0.316	0.360
0.90	0.104	0.146	0.217	0.274	0.324	0.368
0.95	0.141	0.177	0.240	0.294	0.342	0.386

Table 2 Dimensionless velocity of sound for ammonia $\varepsilon(T_r, x)$

	x					
T_r	0.1	0.2	0.4	0.6	0.8	1.0
0.55	0.075	0.117	0.177	0.221	0.259	0.291
0.60	0.077	0.123	0.188	0.237	0.278	0.313
0.65	0.078	0.127	0.196	0.249	0.293	0.331
0.70	0.080	0.129	0.202	0.257	0.304	0.344
0.75	0.082	0.132	0.206	0.264	0.312	0.354
0.80	0.087	0.135	0.210	0.269	0.319	0.362
0.85	0.097	0.143	0.217	0.276	0.326	0.371
0.90	0.117	0.159	0.228	0.286	0.337	0.381
0.95	0.156	0.190	0.251	0.304	0.351	0.394

Table 3 Dimensionless velocity of sound for Freon-12 $\varepsilon(T_r, x)$

	x					
T_r	0.1	0.2	0.4	0.6	0.8	1.0
0.55	0.056	0.096	0.156	0.202	0.240	0.274
0.60	0.061	0.104	0.168	0.218	0.260	0.296
0.65	0.066	0.113	0.183	0.237	0.283	0.323
0.70	0.073	0.121	0.196	0.256	0.304	0.347
0.75	0.079	0.128	0.206	0.267	0.318	0.363
0.80	0.085	0.133	0.209	0.271	0.320	0.367
0.85	0.092	0.136	0.210	0.271	0.322	0.369
0.90	0.105	0.143	0.211	0.272	0.323	0.369
0.95	0.136	0.167	0.226	0.279	0.329	0.374

Table 4 Dimensionless velocity of sound for isobutane $\varepsilon(T_r, x)$

	x					
T_r	0.1	0.2	0.4	0.6	0.8	1.0
0.55	0.068	0.113	0.176	0.223	0.263	0.289
0.60	0.069	0.116	0.184	0.236	0.280	0.318
0.65	0.069	0.117	0.189	0.245	0.292	0.333
0.70	0.069	0.116	0.190	0.249	0.299	0.342
0.75	0.069	0.115	0.190	0.250	0.301	0.346
0.80	0.072	0.115	0.188	0.248	0.301	0.347
0.85	0.078	0.118	0.187	0.246	0.298	0.345
0.90	0.091	0.126	0.189	0.245	0.296	0.342
0.95	0.117	0.146	0.199	0.248	0.294	0.337

Table 5 Average and standard deviation* of $\varepsilon(T_r, x)$

	x					
T_r	0.1	0.2	0.4	0.6	0.8	1.0
0.55	0.068	0.110	0.170	0.215	0.253	0.286
	(0.0076)	(0.0083)	(0.008)	(0.008)	(0.0088)	(0.0091)
0.60	0.071	0.116	0.181	0.230	0.272	0.307
	(0.0066)	(0.0073)	(0.0075)	(0.0076)	(0.008)	(0.0087)
0.65	0.074	0.121	0.190	0.243	0.288	0.327
	(0.0054)	(0.0057)	(0.0046)	(0.0045)	(0.0048)	(0.0057)
0.70	0.076	0.124	0.199	0.253	0.300	0.342
	(0.0047)	(0.0053)	(0.0059)	(0.0032)	(0.0038)	(0.0052)
0.75	0.078	0.126	0.207	0.260	0.309	0.352
	(0.0052)	(0.0066)	(0.0065)	(0.0065)	(0.0067)	(0.0075)
0.80	0.082	0.129	0.203	0.263	0.314	0.358
	(0.0059)	(0.0080)	(0.0089)	(0.0095)	(0.0084)	(0.0084)
0.85	0.088	0.133	0.206	0.264	0.315	0.361
	(0.0069)	(0.0093)	(0.0112)	(0.0113)	(0.0107)	(0.0103)
0.90	0.104	0.144	0.211	0.266	0.319	0.365
	(0.0092)	(0.012)	(0.014)	(0.0124)	(0.015)	(0.014)
0.95	0.138	0.170	0.229	0.281	0.329	0.368
	(0.0139)	(0.016)	(0.0194)	(0.0212)	(0.0216)	(0.0204)

* In parentheses

Random calculations for carbon dioxide and oxygen confirm this observation; this indicates that this behaviour of the two-phase velocity of sound is general for evaporating mixtures.

Critical flux and the velocity of sound

The critical flux of a two-phase mixture is an important property which is useful in calculations for the maximum mass flux through ducts, nozzles or leakages[9,18,19]. In single-phase flow, the velocity of sound and the critical mass flux are directly linked by the relationship:

$$G = \alpha \rho \tag{18}$$

It seems, however, that this is not the case for two-phase mixtures, where the two quantities are not related, except for the states of high quality. This is shown graphically in Figs 2(a), (b) and (c) where the fractional difference between the critical velocity, $V_{cr}(=G/\rho)$, and the velocity of sound is shown against temperature for different qualities. V_{cr} is deduced from experimental data for water[20,21] assuming homogeneous density for the mixture.

It may be observed that it is only for the high quality case ($x=0.8$) that this fractional difference is almost zero. Otherwise, the fractional difference becomes substantial and there is no apparent way to deduce V_{cr} from α. It is thought that this discrepancy is due to the inhomogeneity of low quality flowing two-phase mixtures. Distinct flow patterns are observed and the two phases have different average velocities. All these phenomena violate the assumptions for the sound velocity computations; the discrepancy observed is expected and contemplated by some authors[9]. The flowing mixture satisfies the homogeneity assumption better at high dryness fractions where α and V_{cr} appear to be the same. However, it must be recalled that the two are defined differently in the two-phase mixtures, the one by Eq (2) and the other in terms of the maximum mass flux. The only conclusion that can be drawn from Fig 2 is that $\alpha(T, x)$ is always less than V_{cr}.

Conclusions

An equation is developed, from thermodynamic considerations, which yields the two-phase velocity of sound analytically. This equation enables us to compute the velocity of sound without graphical estimations from diagrams. The functional form of this equation is such as to yield the velocity of sound as a state property of the mixture, whose thermodynamic coordinates are T and x. This function is computed for water, ammonia, Freon-12 and isobutane. Other available data for the velocity of sound[1,8] agree well with the computed values.

It seems that there is an apparent similarity for the values of the sound velocity for corresponding states of the four substances examined; random tests with two other substances (CO_2 and O_2) confirm this similarity. The corresponding values of α for all the substances are very close, as shown by the statistical analysis in Table 5, for reasons which are not clear to the authors at the moment. However, there is no apparent relationship between the velocity and the critical velocity.

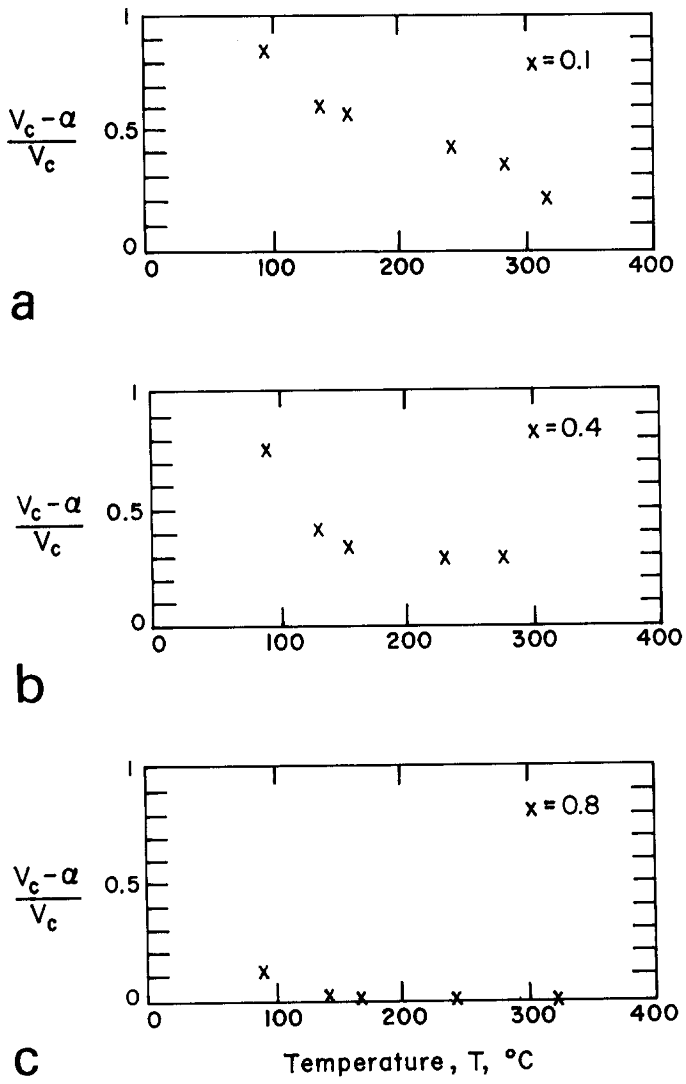

Fig 2 Relative error between critical and sonic velocities

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